

Question 1: RTP May 2023

Ramesh has identified stocks of two companies A and B having good investment potential:

Following data is available for these stocks:

Year	A (Market Price per Share in ₹)	B (Market Price per Share in ₹)
2013	19.60	8.70
2014	18.75	12.80
2015	33.42	16.20
2016	42.64	18.25
2017	43.25	15.60
2018	44.60	13.25
2019	34.75	18.60

You are required to calculate:

- The Risk and Return by investing in Stock A and B
- The Risk and Return by investing in a portfolio of these Stocks if he invests in Stock A and B in proportion of 6 : 4.
- The better opportunity for investment

Solution 1:

Year	A				B				(Return - A) x (Return - B)
	Market Price Per Share in ₹	Return(%)	Return-A	Squared	Market Price Per Share in ₹	Return(%)	Return - B	Squared	
2013	19.60				8.70				
2014	18.75	-4.34	-18.33	335.9889	12.80	47.13	30.94	957.2836	-567.1302
2015	33.42	78.24	64.25	4128.0625	16.20	26.56	10.37	107.5369	666.2725
2016	42.64	27.59	13.60	184.9600	18.25	12.65	-3.54	12.5316	-48.1440
2017	43.25	1.43	-12.56	157.7536	15.60	-14.52	-30.71	943.1041	385.7176
2018	44.60	3.12	-10.87	118.1569	13.25	-15.06	-31.25	976.5625	339.6875
2019	34.75	-22.09	-36.08	1301.7664	18.60	40.38	24.19	585.1561	-872.7752
		83.95		6226.6883			97.14	3582.1748	-96.3718
Mean (A)		13.99	Variance 1037.7814		Mean (B)		16.19	Variance 597.0291	Cov. = -16.0620

(i) Return A = 13.99% and Risk (SD) = $\sqrt{1037.7814}$ = 32.2146 and Return B = 16.19% and Risk (SD) = $\sqrt{597.0291}$ = 24.4342

(ii) Return of Portfolio = $0.60 \times 13.99\% + 0.40 \times 16.19\% = 14.87\%$

Risk (Standard Deviation) of Portfolio = $[0.60^2 \times 1037.7814 + 0.40^2 \times 597.0291 + 2 \times 0.60 \times 0.40 \times (-16.0620)]^{1/2}$

= $[373.6013 + 95.5247 - 7.7098]^{1/2} = 21.4806$

(iii) On the basis of Return 'B' is preferable and on the basis of Risk 'Portfolio Investment' is preferable over the individual stocks.

Question 2: MTP 1 May 2023 (March)

Write any four differences between 'Systematic risk' and 'Unsystematic risk'.

Solution 2:

Differences between Systematic Risk and Unsystematic Risk

S. No.	Systematic Risk	Unsystematic Risk
1.	Refers to the variability of return on stocks or portfolio associated with changes in return on the market as a whole.	Refers to risk unique to a particular company or industry.
2.	It arises due to risk factors that affect the overall market such as changes in	It arises due to risk factors that are particular to any company or industry.

	the nation's economy, tax reform by the Government or a change in the world energy situation.	
3.	Since this affects securities overall and consequently, cannot be diversified away.	This risk can be virtually eliminated from a portfolio through diversification.
4.	Beta is a measure of Systematic Risk.	Standard Deviation is a measure of Systematic Risk.

Question 3: MTP 1 May 2023 (March)

A company has a choice of investments between several different equity oriented mutual funds. The company has an amount of ₹ 100 lakhs to invest. The details of the mutual funds are as follows:

Mutual Funds	A	B	C	D	E
Beta	1.5	1.0	0.8	2.0	0.7

PLAN I

If the company invests 20% of its investments in each of the first two mutual funds (A and B) and balance in equal amounts in the mutual funds C, D and E, what is the beta of the portfolio?

PLAN II

If the company invests 15% of its investment in C, 15% in A, 10% in E and the balance in equal amounts in the other two mutual funds, what is the beta of the portfolio?

If the expected return of market portfolio is 12% at a beta factor of 1.0, what will be the expected return on the portfolio in both the plans given above?

Solution 3:

Plan I: Investment in A and B at 20 % each and balance in equal proportion in C, D, and E.

Mutual Fund	Proportion of Investment	Beta	Proportion × Fund beta
A	0.2	1.50	0.30
B	0.2	1.00	0.20
C	0.2	0.80	0.16
D	0.2	2.00	0.40
E	0.2	0.70	0.14
Portfolio beta			1.20

Plan II: Investment in A at 15%, C at 15% and E at 10% and balance in equal proportion in B and D:

Mutual Fund	Proportion of Investment	Beta	Proportion × Fund beta
A	0.15	1.50	0.225
B	0.30	1.00	0.300
C	0.15	0.80	0.120
D	0.30	2.00	0.600
E	0.10	0.70	0.070
Portfolio Beta			1.315

Expected return = Market return × Portfolio Beta

Plan	Return
I	$12\% \times 1.20 = 14.40\%$
II	$12\% \times 1.315 = 15.78\%$

Question 4: MTP 2 May 2023 (April)

Shiva has a fund of ₹ 5 lacs which he wants to invest in share market with rebalancing target after every 15 days to start with for a period of one month from now. The present NIFTY is 17025. The minimum NIFTY within a month can at most be 15322.50. He wants to know as to how he should rebalance his portfolio under the following situations, according to the theory of Constant Proportion Portfolio Insurance Policy, using "2" as the multiplier:

- (1) Immediately to start with.
- (2) 15 days later-being the 1st day of rebalancing if NIFTY falls to 16321.89.

(3) 15 days further from the above date if the NIFTY touches 17512.14.

Note: Assume that the value of his equity component will change in tandem with that of the NIFTY.

Solution 4:

Maximum decline in one month = $\frac{17025-15322.50}{17025} \times 100 = 10\%$

(1) Immediately to start with

Investment in equity = Multiplier x (Portfolio value – Floor value)

= 2 (5,00,000 – 4,50,000) = ₹ 1,00,000

Shiva may invest ₹ 1,00,000 in equity and balance in risk free securities.

(2) After 15 days

Value of equity = 1,00,000 x 16321.89 / 17025 = ₹ 95,870

Value of risk free investment = ₹ 4,00,000

Total value of portfolio = ₹ 4,95,870

Investment in equity = Multiplier x (Portfolio value – Floor value)

= 2 (4,95,870 – 4,50,000) = ₹ 91,740

Revised Portfolio:

Equity = ₹ 91,740

Risk free Securities = ₹ 4,95,870 – ₹ 91,740 = ₹ 4,04,130

(3) After another 15 days

Value of equity = 91,740 x 17512.14 / 16321.89 = ₹ 98,430

Value of risk free investment = ₹ 4,04,130

Total value of portfolio = ₹ 5,02,560

Investment in equity = Multiplier x (Portfolio value – Floor value)

= 2 (5,02,560 – 4,50,000) = ₹ 1,05,120

Revised Portfolio:

Equity = ₹ 1,05,120

Risk Free Securities = ₹ 5,02,560 – ₹ 1,05,120 = ₹ 3,97,440

The investor should off-load ₹ 6,690 of risk free securities and divert to Equity.

Question 5: MTP 2 May 2023 (April)

On the basis of given information, Mr. XLY want to create a portfolio equally as risky as the market and is having ₹ 20,00,000 to invest.

Assets	Investment	Beta
Stock A	₹ 4,00,000	0.70
Stock B	₹ 5,00,000	1.10
Stock C	?	1.60
Debenture (D)	?	0

How do you recommend and interpret the risk scenario and investment in all the securities?

Solution 5:

Let WA, WB, WC and WD be the weights of Stock A, B, C and Debenture respectively.

WA = 4,00,000 ÷ 20,00,000 = 0.20

WB = 5,00,000 ÷ 20,00,000 = 0.25

Now = WC + WD = 1 – WA – WB = 0.55

It is given in the question that Portfolio should be as risky as that of the market. It means Beta of the portfolio should be 1.

Hence,

WA (0.7) + WB (1.1) + WC (1.6) + WD (0) = 1

0.2 × 0.7 + 0.25 × 1.1 + 1.6WC + WD × 0 = 1

0.14 + 0.275 + 1.6WC + 0 = 1

1.6WC = 1 – 0.415

1.6 WC = 0.585

W = 0.3656

Weight of Debenture (WD) = 1- 0.2 – 0.25 – 0.3656 = 0.1844

Hence, Amount invested in Stock C = 0.3656 × 20,00,000 = ₹ 7,31,200

Amount invested in Debenture (D)

$$= 0.1844 \times 20,00,000 = ₹ 3,68,800$$

Thus, amount to be invested in Stock (C) is ₹ 7,31,200 and in Debenture is ₹ 3,68,800.

Question 6: MTP 2 May 2023 (April)

Mr. A is holding 1000 shares of face value of ₹ 100 each of M/s. ABC Ltd. He wants to hold these shares for long term and have no intention to sell.

On 1st January 2020, M/s XYZ Ltd. has made short sales of M/s. ABC Ltd.'s shares and approached Mr. A to lend his shares under Stock Lending Scheme with following terms:

- Shares to be borrowed for 3 months from 01-01-2020 to 31-03-2020,
- Lending Charges/Fees of 1% to be paid every month on the closing price of the stock quoted in Stock Exchange and
- Bank Guarantee will be provided as collateral for the value as on 01 -01-2020.

Other Information:

- Cost of Bank Guarantee is 8% per annum,
- On 29-02-2020 M/s ABC Ltd., declared dividend of 25%,
- Closing price of M/s. ABC Ltd.'s share quoted in Stock Exchange on various dates are as follows:

Date	Share Price in Scenario -1 Bullish	Share Price in Scenario -2 Bullish
01-01-2020	1000	1000
31-01-2020	1020	980
29-02-2020	1040	960
31-03-2020	1050	940

You are required to find out:

- Earning of Mr. A through Stock Lending Scheme in both the scenarios,
- Total Earnings of Mr. A during 01-01-2020 to 31-03-2020 in both the scenarios,
- What is the Profit or loss to M/s. XYZ by shorting the shares using through Stock Lending Scheme in both the scenarios?

Solution 6:

Earnings of Mr. A through stock lending scheme

		Scenario 1	Scenario 2
(i)	Lending fee		
	31-01-20 1020 x 1% and 980 x 1%	10.20	9.80
	29-02-20 1040 x 1% and 960 x 1%	10.40	9.60
	31-03-20 1050 x 1% and 940 x 1%	10.50	9.40
	Earnings from lending per Share (A)	31.10	28.80
	Total No. of Shares	1000	1000
	Total Earning from Lending	31,100	28,800
(ii)	Dividend income per Share (B)	25.00	25.00
	Total earnings per share (A) + (B)	56.10	53.80
	Total No. of Shares	1000	1000
	Total Earning	56,100	53,800
(iii)	Gain on shortening the shares		
	(1,050 - 1,000) and (1,000 - 940)	(50.00)	60.00
	Lending fees paid	(31.10)	(28.80)
	Bank guarantee charges @ 8% p.a.	(20.00)	(20.00)
	Gain Per Share	(101.10)	11.20
	Total No. of Shares	1000	1000
	Total Gain on shortening the shares	(1,01,100)	11,200

Question 7: MTP 2 May 2023 (April)

Mr. A owns a portfolio with the following characteristics:

	Security X	Security Y	Risk Free security
Beta	1.60	1.80	0
Expected Return	15%	16%	7%

- If Mr. A has ₹ 2,00,000 to invest and sells short ₹ 1,00,000 of security B and purchases ₹ 3,00,000 of security A what is the sensitivity of Mr. A's portfolio ?

- (ii) If Mr. A borrows ₹ 2,00,000 at the risk free rate and invests the amount he borrows along with the original amount of ₹ 2,00,000 in security X and Y in the same proportion as described in part (i), what is sensitivity of the portfolio?
- (iii) What is the expected market risk premium?

Solution 7:

(i) Mr. A's position in the two securities are +3 in security X and -1 in security Y. Hence the portfolio beta shall be calculated as follows:

$$\text{Sensitivity} = 1.60 \times 300000/200000 - 1.80 \times 100000/200000 = 1.50 \text{ times}$$

(ii) Mr. A's current position: -

$$\text{Security X } ₹ 6,00,000 / ₹ 2,00,000 = 3$$

$$\text{Security Y } -₹ 2,00,000 / ₹ 2,00,000 = -1$$

$$\text{Risk Free Asset } -₹ 2,00,000 / ₹ 2,00,000 = -1 \quad \text{Sensitivity} = 3 \times 1.60 + (-1 \times 1.80) + (-1 \times 0) = 3 \text{ times}$$

(iii) Expected Return = Risk Free Rate of Return + Risk Premium

Let X be the market risk premium,

Accordingly

Using Security X's Return

$$\text{Total Return} = 15\% = 7\% + 1.6X$$

$$\text{Risk Premium (X)} = 8\% / 1.6 = 5\%$$

Alternatively using Security Y's Return

$$\text{Total Return} = 16\% = 7\% + 1.8X$$

$$\text{Risk Premium (X)} = 9\% / 1.8 = 5\%$$

Question 8: MTP 2 May 2023 (April)

Details about portfolio of shares of an investor is as below:

Shares	No. of shares (lakh)	Price per share	Beta
A Ltd.	10.00	₹ 400	1.50
B Ltd.	6.00	₹ 500	1.20
C Ltd.	5.00	₹ 600	1.80

The investor thinks that the risk of portfolio is very high and wants to reduce the portfolio beta to 1.125. He is considering two below mentioned alternative strategies:

- (i) Dispose off a part of his existing portfolio to acquire risk free securities, or
- (ii) Take appropriate position on Nifty Futures which are currently traded at 5000 and each Nifty points is worth ₹ 500.

You are required to determine:

- (1) portfolio beta,
- (2) the value of risk free securities to be acquired,
- (3) the number of shares of each company to be disposed off,
- (4) the number of Nifty contracts to be bought/sold; and
- (5) the value of portfolio beta for 4% rise in Nifty. (10 Marks)

Solution 8:

Shares	No. of shares (lakhs) (1)	Market Price of Per Share (2)	× (2) (₹ lakhs)	% to total (w)	β(x)	wx
A Ltd.	10.00	400.00	4000.00	0.40	1.50	0.60
B Ltd.	6.00	500.00	3000.00	0.30	1.20	0.36
C Ltd.	5.00	600.00	3000.00	0.30	1.80	0.54
			10000.00	1.00		1.50

(1) Portfolio beta 1.50

(2) Required Beta 1.125

Let the proportion of risk free securities for target beta 1.125 = p

$$1.125 = 0 \times p + 1.50 (1 - p)$$

$$p = 0.25 \text{ i.e. } 25\%$$

Shares to be disposed off to reduce beta (10000 × 25%) ₹2,500 lakh and Risk Free securities to be acquired.

(3) Number of shares of each company to be disposed off

Shares	% to total (w)	Proportionate Amount (₹ lakhs)	Market Price Per Share	No. of Shares (Lakh)
A Ltd.	0.40	1000.00	400.00	2.50
B Ltd.	0.30	750.00	500.00	1.50
C Ltd.	0.30	750.00	600.00	1.25

Alternatively, students can also compute above no. of shares by directly multiplying current holding with 25%.

(4) Number of Nifty Contract to be sold

$$\frac{(1.50 - 11.25) \times 10000 \text{ lakh}}{5,000 \times 500} = 150 \text{ contracts}$$

(5) 4% rises in Nifty is accompanied by 4% x 1.50 i.e. 6% rise for portfolio of shares

	₹ Lakh
Current Value of Portfolio of Shares	10000
Value of Portfolio after rise	10600
Mark-to-Market Margin paid on short position (5000 × 0.04 × 500 × 150)	150
Value of the portfolio after rise of Nifty	10450
% change in value of portfolio (10450 – 10000) / 10000	4.50%
% rise in the value of Nifty	4%
Beta	1.50

Question 9: May 2023

Mr. Potential has made investments in two mutual funds. The following information is available:

Mutual Fund	Smart	Growth
Jensen Alpha	1.10%	1.50%
Treynor's Ratio	0.0714	0.0775
Actual Return	8.50%	9.10%
Risk Premium		4%

You are required to calculate:

- Beta (β) for both the funds
- Risk free Rate
- Security Market Line

Solution 9:

	Smart	Growth
Jensen Alpha	0.011	0.015
Treynor's Ratio	0.0714	0.0775
Actual Return	0.085	0.091
Risk Premium	0.04	0.04

$$\text{Jensen Alpha} = \text{Actual Return} - E(r)$$

$$\text{Treynor's Ratio} = \frac{E(r) - R_f}{\beta}$$

$$E(r) = \text{Actual Return} - \text{Jensen Alpha}$$

For Smart:

$$0.04\beta_s = 0.074 - R_f \quad (1)$$

$$0.0714\beta_s = 0.085 - R_f \quad (2)$$

On solving (1) and (2), we get $\beta_s = 0.35$ and $R_f = 0.06$ **For Growth:**

$$0.04\beta_g = 0.076 - R_f \quad (3)$$

$$0.0775\beta_g = 0.091 - R_f \quad (4)$$

On solving (3) and (4), we get $\beta_g = 0.40$ and $R_f = 0.06$

- Beta of Smart Mutual Fund is 0.35 and Growth Mutual Fund is 0.40.
- Risk free Rate = 6%

(iii) Security Market Line for Smart = $0.06 + 0.04\beta$

Security Market Line for Growth = $0.06 + 0.04\beta$

Alternative Solution: In case students have assumed Risk Premium as Equity Risk Premium of respective securities.

Working Notes:

(i) Smart

Jensen Alpha = 0.011

Actual Return = 0.085

Thus, expected return (as per CAPM) = 0.074

Accordingly, Risk Free Rate of Return:

$$0.074 = R_f + 0.04$$

$$R_f = 0.034$$

Treynor's Ratio = 0.0714

$$0.0714 = \frac{0.085 - 0.034}{\beta_s}$$

$$\beta_s = 0.714$$

$$\text{Market Risk Premium} = \frac{0.04}{0.714} =$$

0.056

(ii) Growth

Jensen Alpha = 0.015

Actual Return = 0.091

Thus, expected return (as per CAPM) = 0.076

Accordingly, Risk Free Rate of Return:

$$0.076 = R_f + 0.04$$

$$R_f = 0.036$$

Treynor's Ratio = 0.0775

$$0.0775 = \frac{0.091 - 0.036}{\beta_G}$$

$$\beta_G = 0.710$$

$$\text{Market Risk Premium} = \frac{0.04}{0.710} = 0.056$$

Security Market Line for Smart = $0.034 + 0.056\beta_s$

Security Market Line for Growth = $0.036 + 0.056\beta_G$

Question 10: May 2023

An Investor is proposing to invest ₹ 10,000/- in two Portfolios A and B in the ratio of 3 : 2. The Portfolios have three securities each with following weights :

	W _x	W _y	W _z
Portfolio A	0.30	0.25	0.45
Portfolio B	0.20	0.45	0.35

You are required to

- Calculate the weight of each stock.
- Calculate the amount allocated to Y and Z if half of the funds are allocated to security X.

Solution 10:

(i) Investment committed to each security would be:

	X (₹)	Y (₹)	Z (₹)	Total (₹)
Portfolio A	1,800	1,500	2,700	6,000
Portfolio B	800	1,800	1,400	4,000
Combined Portfolio	2,600	3,300	4,100	10,000
Stock weights	0.26	0.33	0.41	

Alternatively, it can also be computed as follows:

Weight of Security X = $0.30 \times 3/5 + 0.20 \times 2/5 = 0.26$

$$\text{Weight of Security Y} = 0.25 \times 3/5 + 0.45 \times 2/5 = 0.33$$

$$\text{Weight of Security Z} = 0.45 \times 3/5 + 0.35 \times 2/5 = 0.41$$

(ii) The equation of critical line takes the following form:

$$WY = a + bWX$$

Substituting the values of WX & WY from portfolio A and B in above equation, we get

$$0.25 = a + 0.30b, \text{ and}$$

$$0.45 = a + 0.20b$$

Solving above equation we obtain the slope and intercept, $a = 0.85$ and $b = -2$ and thus, the critical line is

$$WY = 0.85 - 2WX$$

If half of the funds is invested in security X then,

$$WY = 0.85 - 1.00 = -0.15$$

$$\text{Since } WX + WY + WZ = 1$$

$$WZ = 1 - 0.50 + 0.15 = 0.65$$

$$\therefore \text{Allocation of funds to Security Y} = -0.15 \times 10,000 = -₹ 1,500 \text{ and}$$

$$\text{Security Z} = 0.65 \times 10,000 = ₹ 6,500$$

Alternatively, it can also be solved as follows:

Amount to be allocated to Y & Z if half of the funds are allocated to X. The balance fund of ₹ 5,000 shall be allocated in the ratio of 33:41.

$$\text{Allocation of funds to - Security Y} = 5,000 \times 33/74 = ₹ 2,230$$

$$\text{Security Z} = 5,000 \times 41/74 = ₹ 2,770$$

Question 11: RTP Nov 2023

John inherited the following securities on his uncle's death:

Types of Security	Nos.	Annual Coupon %	Maturity Years	Yield %
Bond A (₹ 1,000)	10	9	3	12
Bond B (₹ 1,000)	10	10	5	12
Preference shares C (₹ 100)	100	11	*	13*
Preference shares D (₹ 100)	100	12	*	13*

*likelihood of being called at a premium over par. Compute the current value of his uncle's portfolio. PVF @12%

Year	1	2	3	4	5
PVF	0.893	0.797	0.712	0.636	0.567

Note: Round Off Calculations with no decimal points

Solution 11:

Computation of current value of John's portfolio

(i) 10 Nos. Bond A, ₹ 1,000 par value, 9% Bonds maturity 3 years:

	₹
Current value of interest on bond A	
1-3 years: ₹ 900 X Cumulative P.V. @ 12% (1-3 years) = ₹ 900 X 2.402	2,162
Add: Current value of amount received on maturity of Bond A	
End of 3rd year: ₹ 1,000 X 10 X P.V. @ 12% (3rd year) = ₹ 10,000 X 0.712	7,120
	<u>9,282</u>

(ii) 10 Nos. Bond B, ₹ 1,000 par value, 10% Bonds maturity 5 years:

Current value of interest on bond B	
1-5 years: ₹ 1,000 X Cumulative P.V. @ 12% (1-5 years) = ₹ 1,000 X 3.605	3,605
Add: Current value of amount received on maturity of Bond B	
End of 5th year: ₹ 1,000 X 10 X P.V. @ 12% (5th year) = ₹ 10,000 X 0.567	5,670
	<u>9,275</u>

(iii) 100 Preference shares C, ₹ 100 par value, 11% coupon 8,462

$$\frac{11\% \times 100 \text{ Nos.} \times ₹ 100}{13\%} = \frac{1,100}{13\%}$$

(iv)	100 Preference shares D, ₹ 100 par value, 12% coupon	9,231
	$\frac{12\% \times 100 \text{ Nos.} \times ₹ 100}{13\%} = \frac{1,200}{13\%}$	

Total current value of his portfolio [(i) + (ii) + (iii) + (iv)] **36,250**

Question 12: RTP Nov 2023

Ankit has a fund of ₹ 10 lacs which he wants to invest in share market with rebalancing target after every 15 days to start with for a period of one month from now. The present NIFTY is 19679. The minimum NIFTY within a month can at most be 18104.68. He wants to know as to how he should rebalance his portfolio under the following situations, according to the theory of Constant Proportion Portfolio Insurance Policy, using "2" as the multiplier:

- (1) Immediately to start with.
 - (2) 15 days later-being the 1st day of rebalancing if NIFTY falls to 19088.63.
 - (3) 15 days further from the above date if the NIFTY touches 20997.493.
- Note: Assume that the value of his equity component will change in tandem with that of the NIFTY.

Solution 12:

$$\text{Maximum decline in one month} = \frac{19679 - 18104.68}{19679} \times 100 = 8\%$$

- (1) Immediately to start with
Investment in equity = Multiplier x (Portfolio value – Floor value)
= 2 (10,00,000 – 9,20,000) = ₹ 1,60,000
Ankit may invest ₹ 1,60,000 in equity and balance in risk free securities.

- (2) After 15 days

Value of equity = 1,60,000 x 19088.63 / 19679	= ₹ 1,55,200
Value of risk free investment	= ₹ 8,40,000
Total value of portfolio	= ₹ 9,95,200

$$\begin{aligned} \text{Investment in equity} &= \text{Multiplier} \times (\text{Portfolio value} - \text{Floor value}) \\ &= 2 (9,95,200 - 9,20,000) = ₹ 1,50,400 \end{aligned}$$

Revised Portfolio:

Equity	= ₹ 1,50,400
Risk free Securities = ₹ 9,95,200 – ₹ 1,50,400	= ₹ 8,44,800

- (3) After another 15 days
Value of equity = 1,50,400 x 20997.493 / 19088.63 = ₹ 1,65,440
Value of risk free investment = ₹ 8,44,800
Total value of portfolio = ₹ 10,10,240

$$\begin{aligned} \text{Investment in equity} &= \text{Multiplier} \times (\text{Portfolio value} - \text{Floor value}) \\ &= 2 (10,10,240 - 9,20,000) = ₹ 1,80,480 \end{aligned}$$

Revised Portfolio:

Equity	= ₹ 1,80,480
--------	--------------

Risk Free Securities = ₹ 10,10,240 – ₹ 1,80,480	= ₹ 8,29,760
---	--------------

Ankit should off-load ₹ 15,040 of risk free securities and divert to Equity.

Question 13: MTP 1 Nov 2023 (September)

Explain the key features of Alternative Investments that make them different from other investment avenues.

Solution 13:

Though here may be many features of Alternative Investment but following are some common features:

- (i) High Fees – Being a specific nature product the transaction fees are quite on higher side.
- (ii) Limited Historical Rate – The data for historic return and risk is verity limited where data for equity market for more than 100 years is available.

- (iii) Illiquidity – The liquidity of Alternative Investment is not good as next buyer may not be easily available due to limited market.
- (iv) Less Transparency – The level of transparency is not adequate due to limited public information available.
- (v) Extensive Research Required – Due to limited availability of market information extensive analysis is required by the Portfolio Managers.
- (vi) Leveraged Buying – Generally investment in alternative investments is highly leveraged

Question 14: MTP 1 Nov 2023 (September)

Expected returns on two stocks for particular market returns are given in the following table:

Market Return	Aggressive	Defensive
10.50%	6%	13.50%
37.50%	60%	27%

You are required to calculate:

- (1) The Betas of the two stocks.
- (2) Expected return of each stock, if the market return is equally likely to be 10.50% or 37.50%.
- (3) The Security Market Line (SML), if the risk free rate is 11.25% and market return is equally likely to be 10.50% or 37.50%.
- (4) The Alphas of the two stocks.

Solution 14:

- (1) The Betas of two stocks:

$$\text{Aggressive stock} \quad - \quad (60\% - 6\%)/(37.50\% - 10.50\%) = 2$$

$$\text{Defensive stock} \quad - \quad (27\% - 13.50\%)/(37.50\% - 10.50\%) = 0.50$$

Alternatively, it can also be solved by using the Characteristic Line Relationship as follows:

$$R_s = \alpha + \beta R_m$$

Where,

α = Alpha

β = Beta

R_m = Market Return

For Aggressive Stock

$$6\% = \alpha + \beta(10.50\%)$$

$$60\% = \alpha + \beta(37.50\%)$$

$$54\% = \beta(27\%)$$

$$\beta = 2$$

For Defensive Stock

$$13.50\% = \alpha + \beta(10.50\%)$$

$$27\% = \alpha + \beta(37.50\%)$$

$$13.50\% = \beta(27\%)$$

$$\beta = 0.50$$

- (2) Expected returns of the two stocks:-

$$\text{Aggressive stock} \quad - \quad 0.5 \times 6\% + 0.5 \times 60\% = 33\%$$

$$\text{Defensive stock} \quad - \quad 0.5 \times 13.50\% + 0.5 \times 27\% = 20.25\%$$

- (3) Expected return of market portfolio = $0.5 \times 10.50\% + 0.5 \times 37.50\% = 24\%$

$$\text{Market risk premium} = 24\% - 11.25\% = 12.75\%$$

$$\text{SML is, required return} = 11.25\% + \beta_i 12.75\%$$

- (4) $R_s = \alpha + \beta R_m$

For Aggressive Stock

$$33\% = \alpha_A + 2(24\%)$$

$$\alpha_A = -15\%$$

For Defensive Stock

$$20.25\% = \alpha_D + 0.50(24\%)$$

$$\alpha_D = 8.25\%$$

Question 15: MTP 2 Nov 2023 (October)

On January 1, 2013 an investor has a portfolio of 5 shares as given below:

Security	Price	No. of Shares	Beta
A	349.30	5,000	1.15
B	480.50	7,000	0.40
C	593.52	8,000	0.90
D	734.70	10,000	0.95
E	824.85	2,000	0.85

The cost of capital to the investor is 10.5% per annum. You are required to calculate:

- The beta of his portfolio.
- The theoretical value of the NIFTY futures for February 2013.
- The number of contracts of NIFTY the investor needs to sell to get a full hedge until February for his portfolio if the current value of NIFTY is 5900 and NIFTY futures have a minimum trade lot requirement of 200 units. Assume that the futures are trading at their fair value.
- The number of future contracts the investor should trade if he desires to reduce the beta of his portfolios to 0.6.

No. of days in a year be treated as 365.

Given: $\ln(1.105) = 0.0998$ and $e(0.015858) = 1.01598$

Solution 15:**(i) Calculation of Portfolio Beta**

Security	Price of the Stock	No. of shares	Value	Weightage w_i	Beta B_i	Weighted Beta
A	349.30	5,000	17,46,500	0.093	1.15	0.107
B	480.50	7,000	33,63,500	0.178	0.40	0.071
C	593.52	8,000	47,48,160	0.252	0.90	0.227
D	734.70	10,000	73,47,000	0.390	0.95	0.370
E	824.85	2,000	16,49,700	0.087	0.85	0.074
			1,88,54,860			0.849

Portfolio Beta = 0.849

(ii) Calculation of Theoretical Value of Future Contract

Cost of Capital = 10.5% p.a. Accordingly, the Continuously Compounded Rate of Interest $\ln(1.105) = 0.0998$

For February 2013 contract, $t = 58/365 = 0.1589$ Further $F = S e^{rt}$

$$F = ₹ 5,900 e^{(0.0998)(0.1589)}$$

$$F = ₹ 5,900 e^{0.015858}$$

$$F = ₹ 5,900 \times 1.01598 = ₹ 5,994.28$$

Alternatively, it can also be taken as follows:

$$= ₹ 5900 e^{0.105 \times 58/365}$$

$$= ₹ 5900 e^{0.01668}$$

$$= ₹ 5900 \times 1.01682 = ₹ 5,999.24$$

(iii) When total portfolio is to be hedged:

$$= \frac{\text{Value of Spot Position requiring hedging}}{\text{Value of Future Contract}} \times \text{Portfolio Beta}$$

$$= \frac{1,88,54,860}{5994.28 \times 200} \times 0.849 = 13.35 \text{ contracts say 13 or 14 contracts}$$

(iv) When total portfolio beta is to be reduced to 0.6:

$$\text{Number of Contracts to be sold} = \frac{P(\beta_p - \beta_p^*)}{F}$$

$$= \frac{1,88,54,860(0.849 - 0.600)}{5994.28 \times 200} = 3.92 \text{ contracts say 4 contracts}$$

Question 16: MTP 2 Nov 2023 (October)

Indira has a fund of ₹ 3 lacs which she wants to invest in share market with rebalancing target after every 10 days to start with for a period of one month from now. She has 3 close friends who have advised following different strategies:

- Buy and Hold strategy

(ii) Constant Ratio

(iii) CPPI

Suppose she immediately starts with investment in Bonds (non-fluctuating) and Equity and decides to rebalance her portfolio after each 10 days and to invest in Nifty as equity component changes in tandem with that of Nifty. Further, Bond has no Beta.

As on date (i.e. beginning of month) Nifty is 5326 and minimum Nifty within a month can be most be 4793.40. If she chooses CPPI she will use "2" as the multiplier. If she chooses Constant Ratio plan she will maintain the ratio of 60:40 in Equity and Bonds respectively. Further, portfolio will be rebalanced each time Nifty is changed by 5% as compared to the previous Nifty.

You are required to evaluate Portfolio Position of Indira under each of the Strategies suggested by her friends and highlight the course of action to be taken if in the coming month after a gap of 10 days Nifty happened:

(1) 10 days later-being the 1st day of rebalancing if NIFTY falls to 5122.96.

(2) 10 days further from the above date if the NIFTY touches 5539.04.

Solution 16:

(i) Under Buy and Hold Strategy

(1) Maximum decline in one month = $\frac{5326-4793.40}{5326} \times 100 = 10\%$

Floor Value = $3,00,000 \times (1 - 0.10) = ₹ 2,70,000$

Under this strategy investor invests an amount equal to floor value in non-fluctuating assets (Bonds). Thus, Indira will invest ₹2,70,000 in Bonds and the remaining ₹30,000 in equity.

(2) After 10 days

Value of equity = $30,000 \times 5122.96/5326 = ₹ 28,856$

Value of Bonds = ₹ 2,70,000

Total Value of Portfolio = ₹ 2,98,856

(3) After another 10 days

Value of equity = $28,856 \times 5539.04/5122.96 = ₹ 31,200$

Value of Bonds = ₹ 2,70,000

Total value of portfolio = ₹ 3,01,200

(ii) Under CPPI

(1) Immediately to start with

Investment in equity = Multiplier $\times$ (Portfolio value – Floor value)

= $2 (3,00,000 - 2,70,000) = ₹ 60,000$

Indira may invest ₹ 60,000 in equity and balance in bonds.

(2) After 10 days

Value of equity = $60,000 \times 5122.96/5326 = ₹ 57,713$

Value of bonds = ₹ 2,40,000

Total value of portfolio = ₹ 2,97,713

Change in Investment (%) = $\frac{(60,000-57,713)}{60,000} = 3.81\%$

As the change in investment is less than 5%, Indira will not rebalance the portfolio.

(3) After another 10 days

Value of equity = $57,713 \times 5539.04/5122.96 = ₹ 62,400$

Value of bonds = ₹ 2,40,000

Total value of portfolio = ₹ 3,02,000

Change in Investment (%) = $\frac{(62,400-57,713)}{57,713} = 8.12\%$

As the change in investment is more than 5%, Indira will rebalance the portfolio.

Investment in equity = Multiplier $\times$ (Portfolio value – Floor value)

= $2(3,02,400-2,70,000) = ₹ 64,800$

Revised Portfolio:

Equity = ₹ 64,800

Bonds = ₹ 3,02,400 – ₹ 64,800 = ₹ 2,37,600

The investor should off-load ₹ 2,400 of bonds and divert to Equity.

(iii) Under Constant Ratio Strategy

(1) The ratio to be maintained is given as 60:40. Thus, Indira will invest ₹ 1,80,000 in equity and ₹ 120,000 in bonds.

(2) After 10 days

Value of equity = $1,80,000 \times 5122.96/5326$	=	₹ 1,73,138
Value of Bonds		₹ 1,20,000
Total value of portfolio	=	₹ 2,93,138

$$\text{Change in Investment (\%)} = \frac{(1,80,000 - 1,73,138)}{1,80,000} = 3.81\%$$

As the change in investment is less than 5%, Indira will not rebalance the portfolio.

(3) After another 10 days

Value of equity = $1,73,138 \times 5539.04/5122.96$	=	₹ 1,87,200
Value of Bonds		₹ 1,20,000
Total value of portfolio	=	₹ 3,07,200

$$\text{Change in Investment (\%)} = \frac{(1,87,200 - 1,73,138)}{1,73,138} = 8.12\%$$

As the change in investment is more than 5%, Indira will rebalance the portfolio.

Revised Portfolio:

$$\text{Equity} = \frac{60}{60+40} \times 3,07,200 = ₹ 1,84,320$$

$$\text{Bonds} = ₹ 3,07,200 - ₹ 1,84,320 = ₹ 1,22,880$$

The investor should off-load ₹ 2,880 of Equity and divert to Bonds.

Question 17: Nov 2023

An investor has categorized all the available stock in the market into the following types and the estimated weights of the categories of stocks in the market index are given below. Further, the sensitivity of returns of these categories of stocks to two factors Inflation and Stock market are also given below:

Category	Weight in Market Index	Factor 1 (Inflation)			Factor 2 (Stock Market)		
		Beta 1	Expected Value in %	Actual Value in %	Beta 2	Expected Value in %	Actual Value in %
Small Cap	20%	1.20	6.70	6.70	0.80	10.00	10.50
Medium Cap	30%	1.75	4.50	6.00	0.90	7.00	8.00
Large Cap	15%	1.30	6.75	8.00	1.165	9.00	10.00
Flexi Cap	35%	1.70	7.00	6.50	0.85	8.85	9.75

Risk Free Rate of Interest is 7.50%. Round off to 2 decimal.

You are required to calculate:

- Expected return on the market index for both the factors.
- Expected return on the market index under Arbitrage Pricing Theory (Existing Scenario).
- Expected return on the market index under Arbitrage Pricing Theory if the composition of the Portfolio is changed to 25% equally in all four categories.
- Which alternative (Existing or Changed) will be more profitable?

Solution 17:

(i) **Expected Return on Market Index for Both factors**

Factor 1

$$= 0.20 \times 6.70\% + 0.30 \times 4.50\% + 0.15 \times 6.75\% + 0.35 \times 7.00\%$$

$$= 1.34\% + 1.35\% + 1.01\% + 2.45\% = 6.15\%$$

Factor 2

$$= 0.20 \times 10\% + 0.30 \times 7\% + 0.15 \times 9\% + 0.35 \times 8.85\%$$

$$= 2\% + 2.10\% + 1.35\% + 3.10\%$$

$$= 8.55\%$$

(ii) **Calculation of expected Return on the Market index under Arbitrage Pricing Theory (Existing Scenario):**

Category	Beta	Factor 1 (Inflation)			
		Actual value	Expected value	Difference	Beta x Diff.
	(a)	(b) (%)	(c) (%)	(b) - (c) = (d) (%)	(e)
Small Cap	1.20	6.70	6.70	0.00	0.00
Medium Cap	1.75	6.00	4.50	1.50	2.63
Large Cap	1.30	8.00	6.75	1.25	1.63
Flexi cap	1.70	6.50	7.00	(0.50)	(0.85)

Factor 2 (Stock Market)					
-------------------------	--	--	--	--	--

Category	Beta	Actual value	Expected value	Difference	Beta x Diff.	Total
	(f)	(g) (%)	(h) (%)	(g) – (h) = (i) (%)	(j)	(e) + (j) = (k)
Small Cap	0.80	10.50	10.00	0.50	0.40	0.40
Medium Cap	0.90	8.00	7.00	1.00	0.90	3.53
Large Cap	1.165	10.00	9.00	1.00	1.17	2.80
Flexi cap	0.85	9.75	8.85	0.90	0.77	(0.08)

Category	Weight in market index (1)	Total Beta x Diff (2)	Expected Return (2 x 1 = 3)
Small Cap	20%	0.40	0.08
Medium Cap	30%	3.53	1.06
Large Cap	15%	2.80	0.42
Flexi cap	35%	(0.08)	(0.03)
Total			1.53
Add: Risk Free Rate of Interest			7.50
Expected Return (%)			9.03

(iii) Expected Return on the Market Index under Arbitrage Pricing Theory under changed scenario:

Category	Weight in market index (1)	Total Beta x Diff (2)	Expected Return (2 x 1 = 3)
Small Cap	25%	0.40	0.10
Medium Cap	25%	3.53	0.88
Large Cap	25%	2.80	0.70
Flexi cap	25%	(0.08)	(0.02)
Total			1.66
Add: Risk Free Rate of Interest			7.50
Expected Return (%)			9.16

(iv) As per the above calculation, the investors by investing 25% equally in all four categories, is profitable compared to the existing composition. As the proposed composition gives rate of return of 9.16% per annum when compared to the existing return of the present portfolio which is 9.03%.

Question 18: Nov 2023

Expected returns on two stocks against BSE SENSEX returns are given in the following table under two scenarios-bullish and bearish:

Market return	Scenario -1: Bullish Case	Scenario -2: Bearish Case
BSE Sensex	25%	-5%
Stock R	32%	-4%
Stock Z	18%	-3%

You are required to calculate:

- The Betas of two stocks R and Z.
- Expected return on each stock, if the likelihood of market achieving Scenario-1 is thrice the likelihood of the market achieving Scenario-2.
- The Security Market Line (SML), if the risk free rate is 8% and likelihood of the market return achieving the bullish base returns of 25% is thrice that of achieving -5% returns.
- The Alphas of the two stocks based on Sharpe Index Model.

Solution 18:

(i) The Betas of two stocks:

$$\text{Stock R} \quad - \quad (32\% + 4\%)/(25\% + 5\%) = 1.2$$

$$\text{Stock Z} \quad - \quad (18\% + 3\%)/(25\% + 5\%) = 0.70$$

Alternatively, it can also be solved by using the Characteristic Line Relationship as follows:

$$R_s = \alpha + \beta R_m$$

Where

α = Alpha

β = Beta

R_m = Market Return

For Stock R

$$32\% = \alpha + \beta(25\%)$$

$$-4\% = \alpha + \beta(-5\%)$$

$$36\% = \beta(30\%)$$

$$\beta = 1.2$$

For Stock Z

$$18\% = \alpha + \beta(25\%)$$

$$-3\% = \alpha + \beta(-5\%)$$

$$21\% = \beta(30\%)$$

$$\beta = 0.70$$

Alternatively, Beta can also be calculated as follows:

Basic Calculation for stock R

(RR)	RR	RR - RR	(RR - RR) ²	(R _m)	RM	R _m - RM	(R _m - RM) ²	(RR - RR) (R _m - RM)
32%	14%	18%	324	25%	10%	15%	225	270
-4%	14%	-18%	324	-5%	10%	-15%	225	270
Total			648				450	540

Basic Calculation for stock Z

(RZ)	RZ	RZ - RZ	(RZ - RZ) ²	(R _m)	RM	R _m - RM	(R _m - RM) ²	(RZ - RZ) (R _m - RM)
18%	7.5%	10.5%	110.25	25%	10%	15%	225	157.50
-3%	7.5%	-10.5%	110.25	-5%	10%	-15%	225	157.50
Total			220.50				450	315

$$\text{Co-Variance (R, M)} = 540/2 = 270$$

$$\text{Co-Variance (Z, M)} = 315/2 = 157.50$$

$$(\sigma M)^2 = 450/2 = 225$$

Beta of stocks R & Z

$$\text{Beta (R)} = \frac{\text{Cov.}(R,M)}{\sigma^2 M} = 270/225 = 1.2$$

$$\text{Beta (Z)} = \frac{\text{Cov.}(R,Z)}{\sigma^2 M} = 157.5/225 = 0.7$$

(ii) Expected returns of the two stocks:

$$\text{Stock R} = 0.75 \times 32\% - 0.25 \times 4\% = 23\%$$

$$\text{Stock Z} = 0.75 \times 18\% - 0.25 \times 3\% = 12.75\%$$

$$\text{(iii) Expected return of market portfolio} = 0.75 \times 25\% + 0.25 \times (-5\%) = 17.50\%$$

$$\text{Market risk prem.} = 17.50\% - 8.00\% = 9.5\%$$

$$\text{SML is, required return} = 8.00\% + \beta i 9.5\%$$

(iv) Alpha for two stocks

Required Return for Stock R

$$E(R) = \alpha_R + \beta_{RM}$$

Accordingly

$$23\% = \alpha_R + 1.20 \times 17.50\%$$

$$\alpha_R = 2\%$$

Required Return for Stock Z

$$E(Z) = \alpha_Z + \beta_{RM}$$

Accordingly

$$12.75\% = \alpha_Z + 0.70 \times 17.50\%$$

$$\alpha_Z = 0.5\%$$

Question 19: RTP May 2024

An investor has decided to invest ₹ 1,00,000 in the shares of two companies, namely, ABC and XYZ. The projections of returns from the shares of the two companies along with their probabilities are as follows:

Probability	ABC (%)	XYZ (%)
0.20	12	16
0.25	14	10
0.25	-7	28
0.30	28	-2

Required:

- Comment on return and risk of investment in individual shares.
- Compare the risk and return of these two shares with a Portfolio of these shares in equal proportions.
- Advise the proportion of each of the above shares to formulate a minimum risk portfolio.

Solution 19:

- Since risk per unit return of D is more than K, hence investor shall choose K Mutual fund from the perspective of risk per unit of return.

Probability	ABC (%)	XYZ (%)	1X2 (%)	1X3 (%)
(1)	(2)	(3)	(4)	(5)
0.20	12	16	2.40	3.20
0.25	14	10	3.50	2.50
0.25	-7	28	-1.75	7.00
0.30	28	-2	8.40	-0.60
Average return			12.55	12.10

Hence the expected return from ABC = 12.55% and XYZ is 12.10%

Probability	(ABC-ABC)	(ABC-ABC) ²	1X3	(XYZ-XYZ)	(XYZ-XYZ) ²	(1)X(6)
(1)	(2)	(3)	(4)	(5)	(6)	
0.20	-0.55	0.3025	0.06	3.9	15.21	3.04
0.25	1.45	2.1025	0.53	-2.1	4.41	1.10
0.25	-19.55	382.2025	95.55	15.9	252.81	63.20
0.30	15.45	238.7025	71.61	-14.1	198.81	59.64
			167.75			126.98

$$\sigma_{ABC}^2 = 167.75(\%)^2; \sigma_{ABC} = 12.95\%$$

$$\sigma_{XYZ}^2 = 126.98(\%)^2; \sigma_{XYZ} = 11.27\%$$

- In order to find risk of a portfolio of two shares, the covariance between the two is necessary here.

Probability	(ABC-ABC)	(XYZ-XYZ)	2X3	1X4
(1)	(2)	(3)	(4)	(5)
0.20	-0.55	3.9	-2.145	-0.43
0.25	1.45	-2.1	-3.045	-0.76
0.25	-19.55	15.9	-310.845	-77.71
0.30	15.45	-14.1	-217.845	-65.35
				-144.25

$$\sigma_P^2 = (0.5^2 \times 167.75) + (0.5^2 \times 126.98) + 2 \times (-144.25) \times 0.5 \times 0.5$$

$$\sigma_P^2 = 41.9375 + 31.745 - 72.125$$

$$\sigma_P^2 = 1.5575 \text{ or } 1.56(\%)$$

$$\sigma_P = \sqrt{1.56} = 1.25\%$$

$$E(R_p) = (0.5 \times 12.55) + (0.5 \times 12.10) = 12.325\%$$

Hence, the return is 12.325% with the risk of 1.25% for the portfolio. Thus, the portfolio results in the reduction of risk by the combination of two shares.

(iii) For constructing the minimum risk portfolio, the condition to be satisfied is

$$X_{ABC} = \frac{\sigma_X^2 - r_{AX} \sigma_A \sigma_X}{\sigma_A^2 + \sigma_X^2 - 2 r_{AX} \sigma_A \sigma_X} \text{ or } \frac{\sigma_X^2 - Cov_{AX}}{\sigma_A^2 + \sigma_X^2 - 2 Cov_{AX}}$$

σ_X = Std. Deviation of XYZ

σ_A = Std. Deviation of ABC

r_{AX} = Coefficient of Correlation between XYZ and ABC

Cov.AX = Covariance between XYZ and ABC.

Therefore,

$$\% \text{ ABC} = \frac{126.98 - (-144.25)}{126.98 + 167.75 - [2 \times (-144.25)]} = \frac{271.23}{583.23} = 0.4650 \text{ or } 46.50\%$$

% ABC = 46.50%,

% XYZ = (1 - 0.4650) = 0.5350 = 53.50%

Question 20: MTP 1 May 2024 (March)

Mr. X is interested in investing ₹ 4,00,000 for which he is considering following three alternatives:

- (i) Invest ₹ 4,00,000 in Security A
- (ii) Invest ₹ 4,00,000 in Security B
- (iii) Invest ₹ 2,40,000 in Security A and ₹ 1,60,000 in Security B

Average annual return earned on Security A and Security B is 15% and 14% respectively. Risk free rate of return is 10% and Market Rate of Return is 12%.

Covariance of returns of Security A, Security B and Market portfolio are as follows:

	Security A	Security B	Market
Security A	4.800	4.300	3.370
Security B	4.300	4.250	2.800
Market	3.370	2.800	3.100

On the basis of above information evaluate the following:

- (i) Expected Return of Security A, B and Portfolio.
- (ii) Variance of return of Security A, Security B and Market.
- (iii) Variance and Standard Deviation of Portfolio.
- (iv) Systematic and Unsystematic Risks of Security A, Security B and Portfolio.

Solution 20:

Working Notes:

(1) Beta of each Security

$$\beta = \frac{Cov(\text{Security}, \text{Market})}{\text{Variance of Market}}$$

$$\beta_A = \frac{3.370}{3.100} = 1.087$$

$$\beta_B = \frac{2.800}{3.100} = 0.903$$

$$(2) \text{ Weight of Security A in portfolio} = \frac{2,40,000}{4,00,000} = 0.60$$

$$\text{Weight of Security B in portfolio} = \frac{1,60,000}{4,00,000} = 0.40$$

(3) Portfolio Beta

$$0.60 \times 1.087 + 0.40 \times 0.903 = 1.013$$

(i) Expected Return

$$\text{Security A Return} = 10\% + 1.087(12\% - 10\%) = 12.17\%$$

$$\text{Security B Return} = 10\% + 0.903(12\% - 10\%) = 11.81\%$$

$$\text{Portfolio Return} = 10\% + 1.013(12\% - 10\%) = 12.03\%$$

(ii) Variance of Returns

$$Cor_{i,j} = \frac{Cov(i,j)}{\sigma_i \sigma_j}$$

Accordingly, for Security A

$$1 = \frac{Cov(A,A)}{\sigma_A \sigma_A}$$

$$\sigma_A^2 = 4.800$$

Accordingly, for Security B

$$1 = \frac{Cov(B,B)}{\sigma_B \sigma_B}$$

$$\sigma_B^2 = 4.250$$

Accordingly, for Market Return

$$1 = \frac{Cov(M,M)}{\sigma_M \sigma_M}$$

$$\sigma_M^2 = 3.100$$

Alternatively, by referring diagonally the given Table these values can identified as follows:

$$Variance_A = 4.800$$

$$Variance_B = 4.250$$

$$Variance_M = 3.100$$

(iii) Variance and Standard Deviation of Portfolio Variance

$$\sigma_{AB}^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B Cov_{A,B}$$

$$= (0.60)^2 (4.800) + (0.40)^2 (4.250) + 2(0.60)(0.40)(4.300)$$

$$Variance = 4.472$$

Standard Deviation

$$\sigma_{AB} = \sqrt{4.472} = 2.115$$

(iv) Systematic and Unsystematic Risks of Security A, Security B and Portfolio

$$\text{Systematic Risk} = \beta^2 \sigma_m^2$$

Accordingly,

$$\text{Systematic Risk of Security A} = (1.087)^2 \times 3.10 = 3.663$$

$$\text{Systematic Risk of Security B} = (0.903)^2 \times 3.10 = 2.528$$

$$\text{Systematic Risk of Portfolio} = (1.013)^2 \times 3.10 = 3.181$$

$$\text{Unsystematic Risk} = \text{Total Risk} - \text{Systematic Risk}$$

Accordingly,

$$\text{Unsystematic Risk of Security A} = 4.80 - 3.663 = 1.137$$

$$\text{Unsystematic Risk of Security B} = 4.250 - 2.528 = 1.722$$

$$\text{Unsystematic Risk of Portfolio} = 4.472 - 3.181 = 1.291$$

Question 21: MTP May 2024 (March)

A trader has in its portfolio shares worth ₹ 170 lakhs at current price and cash ₹ 30 lakhs. The beta of the share portfolio is 1.6.

Evaluate:

(i) Current portfolio beta

(ii) Portfolio beta after 3 months if the trader on current date goes for a long position on ₹ 200 lakhs Nifty futures and after 3 months the price of shares dropped by 3.2%.

Solution 21:**(i) Current Portfolio Beta**

$$\begin{aligned}\text{Current Beta for share portfolio} &= 1.6 \\ \text{Beta for cash} &= 0 \\ \text{Current portfolio beta} &= 170/200 \times 1.6 + 0 \times 30/200 = 1.36\end{aligned}$$

(ii) Portfolio beta after 3 months:

$$\text{Beta for portfolio of shares} = \frac{\text{Change in value of portfolio of share}}{\text{Change in value of market portfolio (Index)}}$$

$$1.6 = \frac{0.032}{\text{Change in value of market portfolio (Index)}}$$

$$\text{Change in value of market portfolio (Index)} = (0.032 / 1.6) \times 100 = 2\%$$

Position taken on 100 lakh Nifty futures : Long

$$\text{Value of index after 3 months} = ₹ 200 \text{ lakh} \times (1.00 - 0.02) = ₹ 196 \text{ lakh}$$

$$\text{Mark-to-market paid} = ₹ 4 \text{ lakh}$$

$$\text{Cash balance after payment of mark-to-market} = ₹ 26 \text{ lakh.}$$

$$\text{Value of portfolio after 3 months} = ₹ 170 \text{ lakh} \times (1 - 0.032) + ₹ 26 \text{ lakh} = ₹ 190.56 \text{ lakh}$$

$$\text{Change in value of portfolio} = \frac{200 \text{ lakh} - 190.56 \text{ lakh}}{200 \text{ lakh}} = 4.72\%$$

$$\text{Portfolio beta} = 0.0472/0.02 = 2.36$$

Question 22: MTP 2 May 2024 (April)

Suppose that economy A is growing rapidly, and you are managing a global equity fund and so far you have invested only in developed- country stocks only. Now you have decided to add stocks of economy A to your portfolio. The table below shows the expected rates of return, standard deviations, and correlation coefficients (all estimates are for aggregate stock market of developed countries and stock market of Economy A).

	Developed Country Stocks	Stocks of Economy A
Expected rate of return (annualized percentage)	20	30
Risk [Annualized Standard Deviation (%)]	16	30
Correlation Coefficient (r) between stock of two economies	0.30	

Assuming the risk-free interest rate to be 6%, you are required to determine:

- What percentage of your portfolio should you allocate to stocks of Economy A if you want to increase the expected rate of return on your portfolio by 1%?
- What will be the standard deviation of your portfolio assuming that stocks of Economy A are included in the portfolio as calculated above?
- Also show how well the Fund will be compensated for the risk undertaken due to inclusion of stocks of Economy A in the portfolio?

Solution 22:

(i) Let the weight of stocks of Economy A be expressed as w, then

$$(1 - w) \times 20\% + w \times 30\% = 21\%$$

$$\text{i.e. } w = 0.1 \text{ or } 10\%.$$

(ii) Variance of portfolio shall be:

$$(0.9)^2(0.16)^2 + (0.1)^2(0.30)^2 + 2(0.9)(0.1)(0.16)(0.30)(0.30) = 0.02423$$

$$\text{Standard deviation is } (0.02423)^{1/2} = 0.15565 \text{ or } 15.56\%.$$

(iii) The Sharpe ratio will improve by approximately 0.09, as shown below:

$$\text{Sharpe Ratio} = \frac{\text{Expected Return} - \text{RiskFreeRate of Return}}{\text{Standard Deviation}}$$

$$\text{Investment in stock of developed countries only: } \frac{20 - 6}{16} = 0.875$$

$$\text{Investment with inclusion of stocks of Economy A: } \frac{21 - 6}{15.56} = 0.964$$

Question 23: MTP 2 May 2024 (April)

An investor has two portfolios known to be on minimum variance set for a population of three securities X, Y and Z having below mentioned weights:

	WX	WY	WZ
Portfolio A	0.30	0.40	0.30
Portfolio B	0.20	0.50	0.30

Calculate the weight for each stock for a portfolio constructed by investing ₹ 10,00,000 in portfolio A and ₹ 6,00,000 in portfolio B.

Solution 23:

Investment committed to each security would be:-

	X (₹)	Y (₹)	Z (₹)	Total (₹)
Portfolio A	3,00,000	4,00,000	3,00,000	10,00,000
Portfolio B	1,20,000	3,00,000	1,80,000	6,00,000
Combined Portfolio	4,20,000	7,00,000	4,80,000	16,00,000
Stock weights	0.2625 Or 0.26	0.4375 Or 0.44	0.3000 Or 0.30	

Question 24: MTP 2 May 2024 (April)

Your client is holding the following securities:

Particulars of Securities	Cost	Dividends/ Interest	Market price at the end of holding period	Beta
	₹	₹	₹	
Equity Shares:				
G Ltd.	20,000	1,450	19,600	0.6
S Ltd.	30,000	1,000	30,400	0.8
B Ltd.	28,000	1,400	32,000	0.6
GOI Bonds	72,000	5,060	71,980	0.01

Evaluate:

- Risk free rate of return.
- Expected rate of return of each security (except GOI Bond), using the Capital Asset Pricing Model (CAPM).

- Note: (1) Use weighted average Beta in calculations.
(2) Round off calculations upto 3 decimal points.

Solution 24:

Particulars of Securities	Cost ₹	Market Price	Capital gain	Dividend/ Interest
G Ltd.	20,000	19,600	-400	1,450
S Ltd.	30,000	30,400	400	1,000
B Ltd.	28,000	32,000	4,000	1,400
GOI Bonds	72,000	71,980	-20	5,060
Total	1,50,000	1,53,980	3,980	8,910

- Risk free return [Return on Govt. Security (GOI Bond)]

$$\frac{5,060 + (72,000 - 71,980)}{72,000} = 7\%$$

- Weighted Average of Beta

$$0.6 \times 19,600/1,53,980 + 0.8 \times 30,400/1,53,980 + 0.60 \times 32,000/1,53,980 + 0.01 \times 71,980/1,53,980 \\ = 0.076 + 0.158 + 0.125 + 0.005 = 0.364$$

Average Return on Portfolio

$$(8,910 + 3,980) / 1,50,000 \times 100\% = 8.593\%$$

Market Return

$$8.593\% = 7\% + (R_m - 7\%) \times 0.364$$

$$R_m = 11.376\%$$

Expected Rate of Return for each security is

$$\text{Rate of Return} = R_f + \beta (R_m - R_f)$$

G Ltd.	= 7.000% + 0.6 (11.376% – 7.000%) = 9.626%
S Ltd.	= 7.000% + 0.8 (11.376% – 7.000%) = 10.501%
B Ltd.	= 7.000% + 0.6 (11.376% – 7.000%) = 9.626%

Question 25: May 2024

An investor has decided to invest Rs.1,00,000 in the shares of X Ltd. and Y Ltd. The desired returns from the shares of the two companies along with their probabilities are as follows:

Probability	X Ltd (%)	Y Ltd (%)
0.20	-5	15
0.50	10	25
0.30	15	-10

You are required to:

- Calculate the risk and return of investment in individual shares.
- Compare the risk and return of these two shares with a portfolio of these shares in equal proportions.
- Find out the proportion of each of the above shares to formulate a minimum risk portfolio.

Solution 25:

(i)

Probability	X Ltd. (%)	Y Ltd. (%)	1 X 2 (%)	1 X 3 (%)
(1)	(2)	(3)	(4)	(5)
0.20	-5	15	-1.00	3.00
0.50	10	25	5.00	12.50
0.30	15	-10	4.50	-3.00
Average return			8.50	12.50

Hence the expected return from X Ltd. = 8.50% and Y Ltd. is 12.50%

Probability	(X - $\bar{X}$)	(X - $\bar{X}$) ²	1 X 3	(Y - $\bar{Y}$)	(Y - $\bar{Y}$) ²	(1)X(6)
(1)	(2)	(3)	(4)	(5)	(6)	
0.20	-13.50	182.25	36.45	2.50	6.25	1.25
0.50	1.50	2.25	1.125	12.50	156.25	78.125
0.30	6.50	42.25	12.675	-22.50	506.25	151.875
			50.25			231.25

$$\sigma^2 X = 50.25(\%)^2; \sigma X = 7.09\%$$

$$\sigma^2 Y = 231.25(\%)^2; \sigma Y = 15.21\%$$

(ii) In order to find the risk of a portfolio of two shares, the covariance between the two is necessary here.

$$\sigma^2 P = (0.5^2 \times 50.25) + (0.5^2 \times 231.25) + 2 \times (-41.25) \times 0.5 \times 0.5$$

$$\sigma^2 P = 12.563 + 57.813 - 20.625$$

$$\sigma^2 P = 49.751 \text{ or } 49.75(\%)$$

$$\sigma P = \sqrt{49.75} = 7.053\% \text{ or } 7.05\%$$

$$E(R_p) = (0.5 \times 8.50) + (0.5 \times 12.50) = 10.50\%$$

	Return	Risk	Return to Risk Ratio of CV	Ranking
X Ltd.	8.50%	7.09	1.20	2
Y Ltd.	12.50%	15.21	0.82	3
Portfolio	10.50%	7.05	1.48	1

Risk of the portfolio is reduced by combining two shares.

(iii) For constructing the minimum risk portfolio the condition to be satisfied is

$$Y = \frac{\sigma^2 X - r_{XY} \sigma_X \sigma_Y}{\sigma^2 X + \sigma^2 Y - 2r_{XY} \sigma_X \sigma_Y} \text{ Or } = \frac{\sigma^2 X - COV_{XY}}{\sigma^2 X + \sigma^2 Y - COV_{XY}}$$

σX = Std. Deviation of X Ltd.

σY = Std. Deviation of Y Ltd.

r_{XY} = Coefficient of Correlation between X Ltd. and Y Ltd.

Cov. XY = Covariance between X Ltd. and Y Ltd.

Therefore,

$$\% Y \text{ Ltd.} = \frac{50.25 - (-41.25)}{50.25 + 231.25 - [2 \times (-41.25)]} = \frac{91.50}{364} = 0.2514 \text{ or } 25.14\% \text{ or } 25\%$$

Y Ltd. = 25.14% or 25%

X Ltd. = 74.86% or 75%

Alternatively, it can also be computed as follows:

For constructing the minimum risk portfolio the condition to be satisfied is

$$X = \frac{\sigma^2 Y - r_{XY} \sigma_X \sigma_Y}{\sigma^2 X + \sigma^2 Y - 2r_{XY} \sigma_X \sigma_Y} \text{ Or } = \frac{\sigma^2 Y - COV_{XY}}{\sigma^2 X + \sigma^2 Y - COV_{XY}}$$

σ_X = Std. Deviation of X Ltd.

σ_Y = Std. Deviation of Y Ltd.

r_{XY} = Coefficient of Correlation between X Ltd. and Y Ltd.

Cov. XY = Covariance between X Ltd. and Y Ltd.

Therefore,

$$\% X \text{ Ltd.} = \frac{231.25 - (-41.25)}{50.25 + 231.25 - [2 \times (-41.25)]} = \frac{272.50}{364} = 0.7486$$

or 74.86% or 75%

Y Ltd. = 25.14% or 25%

X Ltd. = 74.86% or 75%

Question 26: MTP 1 Nov 2024 (September)

Mr. A owns a portfolio with the following characteristics:

	Security X	Security Y	Risk Free security
Factor 1 sensitivity (λ_1)	0.80	1.50	0
Factor 2 sensitivity (λ_2)	0.60	1.20	0
Expected Return*	15%	20%	10%

* Generated by a two-factor model. Required:

- Compute the sensitivity of Mr. A's portfolio to the two factors if Mr. A has ₹ 3,00,000 to invest and sells short ₹ 1,50,000 of security Y and purchases ₹ 4,50,000 of security X.
- Compute the sensitivity of the portfolio to the two factors if Mr. A borrows ₹ 3,00,000 at the risk free rate and invests the amount he borrows along with the original amount of ₹ 3,00,000 in security X and Y in the same proportion as described in part (i).
- Suppose Mr. D, one of the friend of Mr. A says the expected return premium of factor 2 is zero. Do you agree with this statement. Substantiate your answer with required calculations.

Solution 26:

(i) Mr. A's position in the two securities are +1.50 in security X and -0.5 in security Y. Hence the portfolio sensitivities to the two factors:-

$$\lambda_1 = 1.50 \times 0.80 + (-0.50 \times 1.50) = 0.45$$

$$\lambda_2 = 1.50 \times 0.60 + (-0.50 \times 1.20) = 0.30$$

(ii) Mr. A's revised position:-

$$\text{Security X} = ₹ 9,00,000 / ₹ 3,00,000 = 3$$

$$\text{Security Y} = ₹ 3,00,000 / ₹ 3,00,000 = -1$$

$$\text{Risk free asset} = ₹ 3,00,000 / ₹ 3,00,000 = -1$$

$$\lambda_1 = 3.0 \times 0.80 + (-1 \times 1.50) + (-1 \times 0) = 0.90$$

$$\lambda_2 = 3.0 \times 0.60 + (-1 \times 1.20) + (-1 \times 0) = 0.60$$

(iii) Expected Return = Risk Free Rate of Return + Risk Premium for each sensitivity factor Accordingly

$$15 = 10 + 0.80 \lambda_1 + 0.60 \lambda_2$$

$$20 = 10 + 1.50 \lambda_1 + 1.20 \lambda_2$$

On solving equation, the value of $\lambda_1 = 0$

Yes, Mr. D is correct in his observation.

Question 27: MTP 2 Nov 2024 (October)

Following are the details of a portfolio consisting of three shares:

Share	Portfolio weight	Beta	Expected return in %	Total variance
A	0.20	0.40	14	0.015
B	0.50	0.50	15	0.025
C	0.30	1.10	21	0.100

Standard Deviation of Market Portfolio Returns = 10% You are given the following additional data:

Covariance (A, B) = 0.030

Covariance (A, C) = 0.020

Covariance (B, C) = 0.040

Calculate the following:

- The Portfolio Beta
- Residual variance of each of the three shares
- Portfolio variance using Sharpe Index Model
- Portfolio variance (on the basis of modern portfolio theory given by Markowitz)

Solution 27:

(i) Portfolio Beta

$$0.20 \times 0.40 + 0.50 \times 0.50 + 0.30 \times 1.10 = 0.66$$

(ii) Residual Variance

To determine Residual Variance first of all we shall compute the Systematic Risk as follows:

$$\beta_A^2 \times \sigma_M^2 = (0.40)^2 (0.01) = 0.0016$$

$$\beta_B^2 \times \sigma_M^2 = (0.50)^2 (0.01) = 0.0025$$

$$\beta_C^2 \times \sigma_M^2 = (1.10)^2 (0.01) = 0.0121$$

Residual Variance

$$A \quad 0.015 - 0.0016 = 0.0134$$

$$B \quad 0.025 - 0.0025 = 0.0225$$

$$C \quad 0.100 - 0.0121 = 0.0879$$

(iii) Portfolio variance using Sharpe Index Model

$$\text{Systematic Variance of Portfolio} = (0.66)^2 \times (0.01) = 0.004356$$

$$\text{Unsystematic Variance of Portfolio} = 0.0134 \times (0.20)^2 + 0.0225 \times (0.50)^2 + 0.0879 \times (0.30)^2 = 0.014072$$

$$\text{Total Variance} = 0.004356 + 0.014072 = 0.018428$$

(iv) Portfolio variance on the basis of Markowitz Theory

$$= (w_A \times w_A \times \sigma_A^2) + (w_A \times w_B \times \text{Cov}_{AB}) + (w_A \times w_C \times \text{Cov}_{AC}) + (w_B \times w_A \times \text{Cov}_{AB}) + (w_B \times w_B \times \sigma_B^2) + (w_B \times w_C \times \text{Cov}_{BC}) + (w_C \times w_A \times \text{Cov}_{CA}) + (w_C \times w_B \times \text{Cov}_{CB}) + (w_C \times w_C \times \sigma_C^2)$$

$$= (0.20 \times 0.20 \times 0.015) + (0.20 \times 0.50 \times 0.030) + (0.20 \times 0.30 \times$$

$$0.020) + (0.20 \times 0.50 \times 0.030) + (0.50 \times 0.50 \times 0.025) + (0.50$$

$$\times 0.30 \times 0.040) + (0.30 \times 0.20 \times 0.020) + (0.30 \times 0.50 \times$$

$$0.040) + (0.30 \times 0.30 \times 0.10)$$

$$= 0.0006 + 0.0030 + 0.0012 + 0.0030 + 0.00625 + 0.0060 + 0.0012 + 0.0060 + 0.0090$$

$$= 0.0363$$

Question 28: Nov 2024

Following is the data regarding Three Securities.

Stock	Expected Return (%)	Std. deviation	Correlation with the Market return
A	19%	2.50	0.840
B	13.50%	2.00	0.540
C	11.00%	0.80	0.975
Market risk		1.20	
Market rate of return	14.00%		
Risk free rate	9.00%		

- Advise which of the above stocks are over, under or correctly valued in the market?
- What strategy would you like to recommend?

Question 29: Nov 2024

On January 1, 2023 an investor has a portfolio of 5 securities as given below:

Security	Price (₹)	No. of shares	Beta
A	612.65	3000	?
B	334.20	5000	1.15
C	454.45	6000	0.40
D	775.10	10000	0.95
E	781.05	3000	0.85

Portfolio beta is 0.859.

The cost of capital to the investor is 10.5% p.a.

You are required to calculate:

- The beta of Security A.
- The theoretical value of the Nifty futures for February, 2023. Current value of Nifty 6500.
- The number of contracts of Nifty the investor needs to sell to get a full hedge until February, for his portfolio, if the current value of Nifty is 6500 and Nifty futures have a minimum trade lot requirement of 200 units. Assume that the Futures are trading at their fair value.
- What will be the new beta if 4 Future contracts are sold to the investors? No. of days in a year be traded as 365 days

Given: $\ln(1.105) = 0.0998$, $e^{0.015858} = 1.01598$ and $e^{0.01668} = 1.01682$

Question 30: Nov 2024

A Portfolio Manager (PM) has three mutual funds in his portfolio. Following are the details of these three mutual funds :

Particulars	Growth fund	Balanced fund	Regular fund	Market
Average Return (%)	7.5	6.3	5.4	
Variance				50.41
Sharpe Ratio	-0.15	-0.36	-0.48	
Treynor's Ratio	-2	-3	-4.80	

The yield on 182 days Treasury bill is 9 per cent per annum.

You are required to calculate

- Variance of the Funds
- Coefficient of Determination of the Funds